two-band antenna matching with stubs

Complete design details for an antenna stub matching system for two harmonically-related amateur bands

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With the sun spot cycle going down there will be greater interest in 40, 80 and 160 meters for DX work in the next few years, bands where rotary beams are difficult if not impracticable. Wire antennas still have their place, and it is desirable, if possible, to build antennas that will have reasonable directional characteristics on at least two bands. For example, the familiar dipole fed with open-wire line operates as two half-waves in phase on the second harmonic and exhibits slight gain in the broadside direction. The popular W8JK array can also be operated on two harmonically-related bands.

The principal problem with the standard arrangement is that the center impedance of such an antenna at the harmonic becomes quite high, on the order of several thousand ohms, and the bandwidth also narrows; that is, the reactance and resistance change is quite large around resonance. Thus, there is a very high vswr on the open-wire line. While high vswr does not result in appreciable *loss* on a good open-wire line, it does lead to problems in maintaining good balance to ground and in minimizing radiation from the transmission line itself.

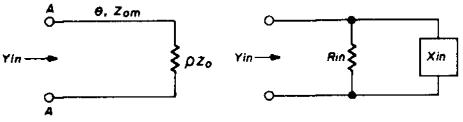
High vswr also complicates the antenna tuner in going from one band to another; depending on the length of feeder, it may be necessary to switch from series tuning on one band to parallel tuning on the other, and band changing becomes complicated. Then, there is the matter of the high voltages and currents along the mismatched line. Even a 600-ohm *flat* line at maximum power will have about 600 to 700 volts rms of rf across it; when the vswr is high this voltage will be appropriately higher. High voltages and currents along a transmission line can cause problems in coupling into other lines (such as telephone lines and tv lead-ins) and in arcing to nearby objects. In short, a low vswr is very desirable, even on an open-wire transmission line.

stub matching

It is possible, using stubs and matching sections, to design an antenna that can be operated on two bands, say 80 and 40 or 160 and 80, so that a reasonable match will be achieved to a specified open-wire line without switching. This particular problem hasn't been treated in the various antenna handbooks where stub matching for only one frequency is discussed.[1]

The analytical approach to this problem can get pretty complicated, and in fact, intractable unless it is done in the right way. The solution lies in using the familiar transmission-line equations in admittance, rather than impedance form, and further, in using half-angle rather than double-angle formulas when going between bands. That is, the electrical length of the matching section and stub is defined at θ at the harmonic frequency, say 7 MHz, and the corresponding length at the fundamental, 3.5 MHz in this case, becomes $\theta/2$.

Fig. 1 gives the basic relationships, the input admittance of a lossless transmission line having characteristic impedance Z_o and length θ. In stub matching, the stub (whether shorted or open) is con-



$$\frac{Z_{om}}{Z_{in}} = Y_{in} = G_{in} + jB_{in} \qquad G_{in} = \frac{Z_{om}}{R_{in}} \qquad B_{in} = \frac{Z_{om}}{X_{in}}$$

$$G_{in} = \rho \, \frac{1 + (\tan\theta)^2}{\rho^2 + (\tan\theta)^2} \qquad (1) \qquad B_{in} = \tan\theta \, \frac{\rho^2 - 1}{\rho^2 + (\tan\theta)^2} \qquad (2)$$

$$\text{at } f \quad G_{in1} = \frac{2\rho 1}{\cos\theta\,(\rho_1{}^2 - 1) + (\rho_1{}^2 + 1)} \qquad (3) \qquad \text{at } f \quad B_{in1} = \frac{\sin\theta\,(\rho_1{}^2 - 1)}{\cos\theta\,(\rho_1{}^2 - 1) + (\rho_1{}^2 + 1)} \qquad (4)$$

$$\text{at } 2f \quad G_{in2} = \frac{\rho_2}{(\cos\theta)^2\,(\rho_2{}^2 - 1) + 1} \qquad (5) \qquad \text{at } 2f \quad B_{in2} = \frac{(\sin\theta)(\cos\theta)(\rho_2{}^2 - 1)}{(\cos\theta)^2\,(\rho_2{}^2 - 1) + 1} \qquad (6)$$

$$\text{where } \rho_1 = \frac{R1}{Z_{om}} \text{ and } \rho_2 = \frac{R2}{Z_{om}}$$

fig. 1. Basic relationships of open-stub impedance matching at two frequencies, f and 2f.

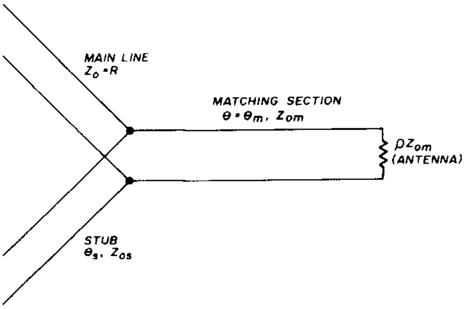


fig. 2. The matching problem.

nected a distance, θ, back from the load or from a known minimum or maximum in the standing wave of the unmatched line. The reactance of the stub is made equal and opposite to the input reactance of the matching section at that point.

For two-band matching the problem is depicted in **fig. 2.** The problem is to find a matching section and stub such that the input impedance with the stub connected, at the point of connection, is resistive and equal to Z_o = R of the main transmission line on each of the two bands. To do this, you must also know the antenna impedance at both frequencies. This is assumed to be resistive, justified on the basis that the antenna will be pruned to resonant length or at least carefully calculated.

Resistance R1 is defined as the antenna resistance at frequency f, and R2 as the antenna resistance at frequency 2f.

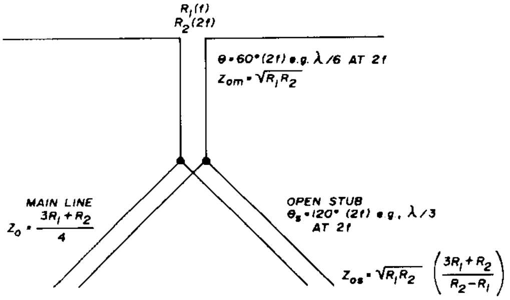


fig. 3. Simplified Johnson match for two-frequency antenna impedance matching.

Using the appropriate trigonometric identities, **eq. 3** through **eq. 6** in **fig. 1** may be derived from **eq. 1** and **eq. 2.** These are the basic relationships for two-band matching. There are four unknowns, Z_{om}, the characteristic impedance of the matching section, Z_{os}, the characteristic impedance of the stub, θ_m, the electrical length of the matching section ($\theta = \theta_m$), and θ_s the electrical length of the stub.

Resistance R1 has previously been *specified* as the antenna resistance at frequency f. R2 is the antenna resistance at 2f. When the reactance has been tuned out by the stub, the resistance R is equal to

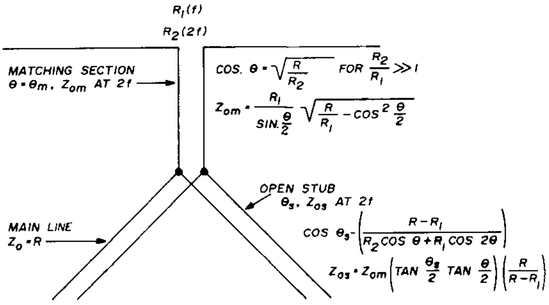


fig. 4. Generalized Johnson match for two bands where R2, the antenna impedance on the higher frequency band, is much greater than R1, the antenna impedance on the lower band, the usual case.

$$R = \frac{Z_{om}}{G_{in1}} = \frac{Z_{om}}{G_{in2}}$$

Where R is the desired match to the main transmission line impedance, Z_o, as shown in **fig. 2.**

special case

Before proceeding to the general case, a most interesting special case is where

$$\rho_1 = \frac{1}{\rho_2} = \rho \text{ or } Z_{om} = \sqrt{R1R2}$$

For this case **eq. 3** is set equal to **eq. 5** to provide the following simple quadratic expression

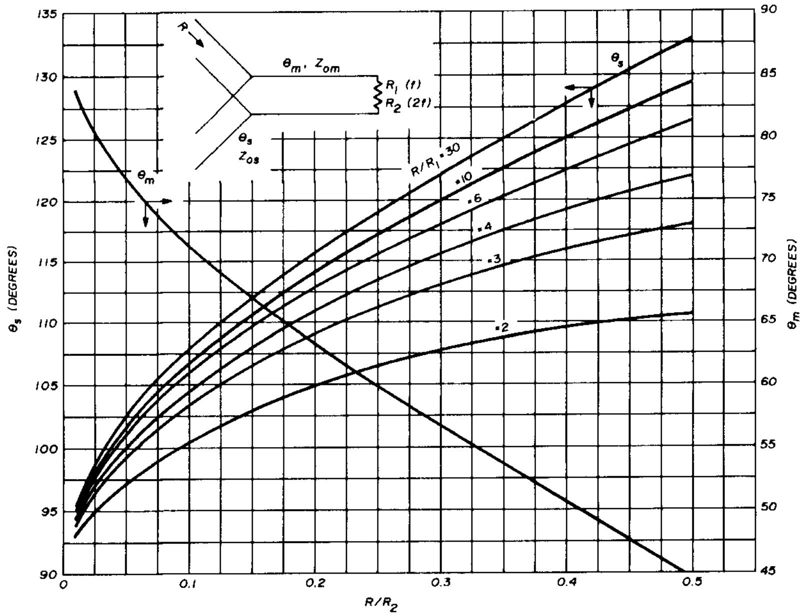


fig. 5. Electrical length of the matching section, θ_m, and the stub, θ_s, at frequency 2f where R2 is much greater than R1.

$$2C^2 + C - 1 = (2C - 1)(C + 1)$$
$$C = \tfrac{1}{2} \text{ or } C = -1$$
where $C = \cos\theta$

For this special case $\theta = 60^\circ$ or 180° at 2f. The latter case is the familiar quarter-wave section at f, half-wave section at 2f. The input impedance at points A-A in **fig. 1** is simply R2 at both frequencies. The problem here is that R2 is usually high, so the main line impedance to match it is unreasonably high. The other case, θ =

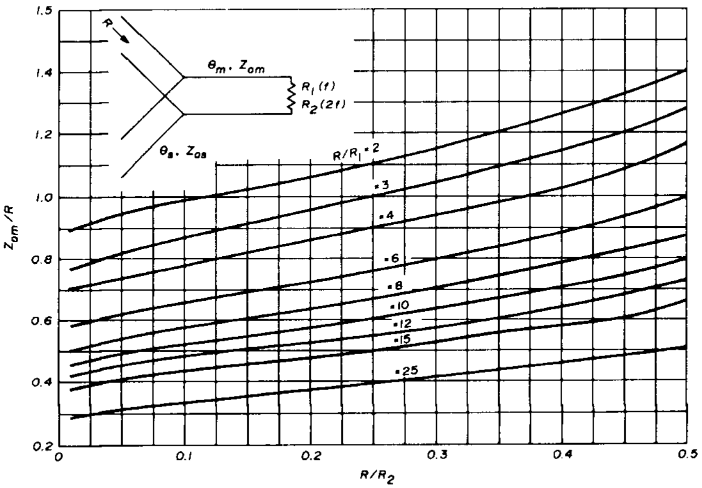


fig. 6. Characteristic impedance of the matching section, Z_{om}, where R2 is much greater than R1.

60° at 2f, is much more useful. For this case the result is

$$R = \frac{3R_1 + R_2}{4} \tag{7}$$

To tune out the reactance in this $\theta = 60°$ case requires an open stub exactly 120° long at 2f with a characteristic impedance of

$$Z_{os} = \sqrt{R_1 R_2}\ \frac{3R_1 + R_2}{R_2 - R_1} \tag{8}$$

The simplified *Johnson Match* is shown in **fig. 3.** Note that the main line characteristic impedance $Z_o = R$ is *not* arbitrary in this case because the quantity Z_{om} has been specified rather than treated as an unknown.

As an example, suppose you had an antenna such that R1 = 80 ohms and R2 = 2000 ohms. Then

$$R = \frac{3 \times 80 + 2000}{4} = 560 \text{ ohms}$$

$$Z_{om} = \sqrt{80 \times 2000} = 400 \text{ ohms}$$

$$Z_{os} = 400\left(\frac{3 \times 80 + 2000}{2000 - 80}\right) = 467 \text{ ohms}$$

$$\theta_m = 60° \text{ at 2f (1/6 wavelength)}$$

$$\theta_s = 120° \text{ at 2f (1/3 wavelength)}$$

You would have a perfect match for a 560-ohm line on both frequencies f and 2f by connecting the two 400-ohm and 467-ohm lines as shown in **fig. 3.**

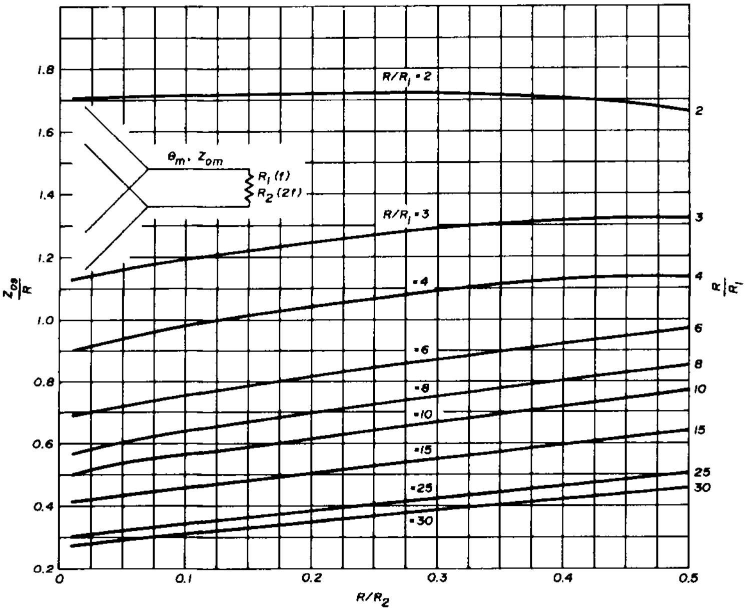


fig. 7. Characteristic impedance of the stub, Z_{os}, where R2 is much greater than R1.

general case

In the special case just considered, you do not have an arbitrary choice of main line characteristic impedance, at least if you want a perfect match. R2 may be considerably higher than the 2000 ohms assumed in the above example, and thus, R may still be unreasonably high, In the general case you first use **eq. 3** and **5** to find Z_{om} and Cos θ (θ and θ_m are used interchangeably here). Then, **eq. 4** and **6** are used together with the stub reactance formula to find θ_s and Z_{os}. The result, when R2 is much larger than R1, the usual case, is given in **fig. 4** as the generalized *Johnson Match.*

To simplify finding the required lengths and characteristic impedances of the line sections, **figs. 5, 6** and **7** have been prepared using the approximate formulas given in **fig. 4** which are valid when R2 is much larger than R1. In **fig. 5** are given the electrical lengths of the matching section and stub in degrees at the harmonic frequency 2f. In **fig. 6** is the Z_{om} family giving the required characteristic impedance of the matching section, and in **fig. 7** is the Z_{os} family giving the required characteristic impedance of the open stub.

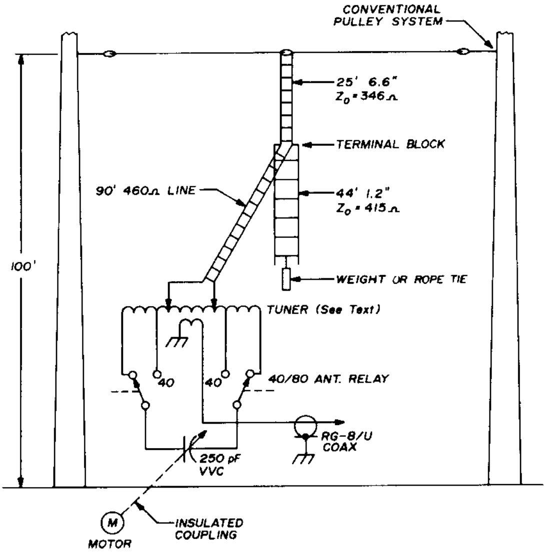


fig. 8. Two-band 3.5- and 7.0-MHz antenna system used at W6MUR incorporates the stub matching system described in the text.

As an example in using these curves, assume an antenna where R2 = 4000 ohms at 2f and R1 = 60 ohms at f. Also assume you want matching on both frequencies to a 600-ohm line, R = 600. Then, compute R/R2 = 0.15 and R/R1 = 10 and enter the curves to find

from **fig. 5**, at R/R2 = .15 and R/R1 = 10, $\theta_m = 67.2^\circ$, $\theta_s = 111^\circ$ at 2f;

from **fig. 6** find Z_{om}/R = 0.55 and compute Z_{om} as 600 x .55 = 330 ohms; and

from **fig. 7** find Z_{os}/R = 0.59 and compute Z_{os} as 600 x .59 = 355 ohms.

antenna input resistance

An excellent, if somewhat obscure, source for accurate information on the center reactance and resistance of antennas in the vicinity of resonance is given in reference 2. This excellent book, by the way, also has sufficient information in graphical form to enable you to design a very good three-element parasitic beam for either best front-to-back ratio or for maximum forward gain (the two do not coincide) without guesswork as to the lengths of driven element, reflector and director. On pages 20 through 25 of this reference will be found some curves giving the center impedance of "nearly half-wave" and "nearly full-wave" center-fed antennas for various conductor thickness. Additionally, the effects of spaced multiple wires are also given.

The effect of ground must also be considered in estimating antenna resistance. With city-lot installations and low heights, this can get pretty indefinite, but if the antenna is reasonably in the clear, curves such as those given by Kraus[3] can be used. It is noted, for example, that the half-wave antenna, when 0.34 wavelength high, has a resistance not of 73 ohms as in free space, but close to 100 ohms. When the half-wave antenna is low, say 0.1 wavelength high, the resistance drops to something like 23 ohms.

test results

The antenna shown in **fig. 8** and **9** has been installed and tested at W6MUR for over two months with excellent results when this article was written. The antenna resistances were estimated for the multiple-wire arrangement at 88 ohms at 3.5 MHz and 2440 ohms at 7.0 MHz, and the matching section and stub were proportioned accordingly. The antenna tuner is a simple parallel-resonant circuit having an edge-wound ribbon coil (surplus) of 20 turns, 4¼ inches in diameter, 8-inches long (about 18 microhenries), tuned by a 250 pF vacuum-variable capacitor driven through an isulated coupling by a reversible gear motor turning a few rpm. The antenna was tapped on to the coil and a fixed link constructed of three turns of number-10 wire inside the coil, to which was connected the 50-ohm coax. The same coil was used on both 3.5 and 7 MHz, with the capacitor tapped down on the coil for 7 MHz by means of an antenna switching relay.

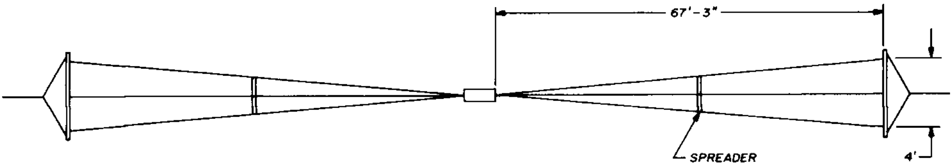
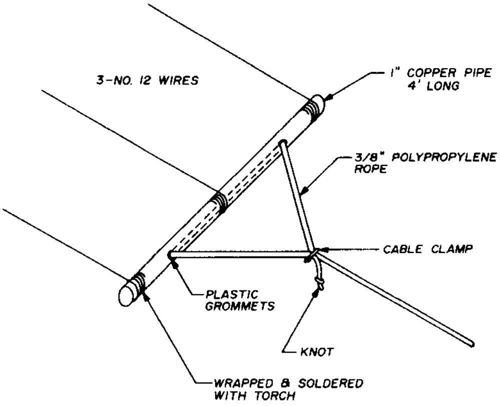


fig. 9. Construction details of the 3.5- and 7.0-MHz antenna used at W6MUR.

Using a grid-dipper signal source and an antenna impedance bridge looking into the link, the points where the antenna feedline was tapped on to the coil were varied until, at resonance, the bridge read 50 ohms at both frequencies. This worked out to be 3 turns between the feeders. The vswr measured into the coaxial line at the transmitter is 1.05:1 at 3500 kHz, 1.6:1 at 3800 kHz, 1.5:1 at 7000 kHz, and 1.9:1 at 7250 kHz. In tuning, once the tuner is very near resonance, you simply watch the reflected power as the motor is energized, and set the variable capacitor for a minimum reading.

This antenna, at 100-feet high, has proved remarkably good for DX work in the preferred direction, which is toward Asia and South America. In the CQ DX contest, November, 1972, on 3.5 MHz 34 countries, 19 zones and 172 stations were worked. The antenna proved itself in the pile-ups for ZD3Z, 9L1GG and 5T5CJ, all worked through eastern-U.S. QRM on 3.5 MHz. Both VU and UL7 have been worked with this antenna on 7-MHz CW, via the long path around sundown.

The main objective of achieving a good match on both bands to the open-wire feedline without excessive switching has been realized. It is hoped that the approach presented here will be useful to others wanting two-band antennas, even if they don't happen to have 135-foot redwood trees to tie them to!



references

1. *Electronic Designer's Handbook,* McGraw-Hill, New York, 1957, page 20-15.
2. Shintaro Uda and Yasuto Mushiake, *Yagi-Uda Antenna,* Sasaki Printing and Publishing Co., Ltd., 27 Tsutsumi-dori, Sendai, Japan, distributed by Maruzen Co., Ltd., 2-chome, Nithombashi-dori, Chuc-ku, Tokyo, Japan, 1954, pages 20-25.
3. John D. Kraus, *Antennas,* McGraw-Hill, New York, 1950, page 305.

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